Area of a Rectangle Proof

The area of a rectangle can be calculated by representing the rectangle on the coordinate plane by bounding it between the x-axis and a function. The function can then be integrated between two endpoints to find the area.

An ideal function to choose is \( f(x) = \text{constant} \) where the constant is the height of the rectangle. The end points of the integral can then be 0, and the length of the rectangle.

Letting \( f(x) = b \), on a graph this looks like the following:

To find the area of the rectangle we formulate the integral \( \int_{0}^{a} f(x) \, dx \)

By solving this integral, we get the formula for calculating the area of a rectangle with length \( a \) and width \( b \):

\[
\int_{0}^{a} f(x) \, dx = \int_{0}^{a} b \, dx = bx\bigg|_{0}^{a} = b(a) - b(0) = ab
\]

Thus we have proved that the area of a rectangle is the length multiplied by the width.