Proving a Continuous Function is Increasing on an Interval

Given a continuous function $f(x)$ it is possible to prove whether or not it is increasing on a given interval $[a,b]$. This is accomplished by examining the first derivative of the function. If $f'(x) > 0$ for all $x \in [a,b]$ then $f(x)$ is increasing on the interval $[a,b]$.

Proof:

By definition $f'(x) = \dfrac{f(b) - f(a)}{b-a}$

If the interval is $[a,b]$ then also by definition $b > a$

Thus the denominator $(b-a) > 0$

Since $f'(x)$ and $(b-a)$ are both positive, then $(f(b) - f(a))$ must also be positive. This only occurs when $f(b) > f(a)$.

Because $f'(x) > 0$ for all $x \in [a,b]$ we can replace a with any arbitrary value c such that $c \in (a,b)$. We can also replace b with any arbitrary value d such that $d \in (c,b)$. We then see that for any c and d that $f(d) > f(c)$. Thus $f(x)$ is strictly increasing on the interval $[a,b]$.

Example:

Prove $f(x) = x^2$ is increasing on the interval $[2,10]$

$f'(x) = 2x$

$f'(x) > 0$ for all $x > 0$ thus $f'(x) > 0$ for all $x \in [2,10]$

We know that $f'(x) = \dfrac{f(b) - f(a)}{b-a}$ and $f'(x) > 0$ for all $x \in [2,10]$, therefore $f(x)$ is strictly increasing on the interval $[2,10]$